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**Mathematical Analysis of Reliability of M Out Of N Warm Stand by System with Repair Facilities**

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**ABSTRACT**

*The present paper is investigated the Mathematical Analysis of Reliability of M Out of N Warm Stand by System with Repair Facilities. Cold stand by and hot standby reliability systems are derived as particular cases of the model. Here we are trying to use Rayleigh distribution to obtain equation of reliability. Reliability discussed with table and graphically*

**Keywords:** Reliability; Stand by System; Availability.

**1.0 Introduction**

Practical systems like power station coal transmission system and wireless communication networks can be modeled as multistage systems these systems can perform their intended functions at full capacity, different level of reduced capacity and of course they can also be totally failed. The multistate system models can represent equipment condition with more accuracy and flexibilities than the binary system models, where the equipments and the system can also be in two possible states, working or failed.  $K$  out of  $N$  system model is generalization system reliability model and has also been studied extensively in the binary context, where the system and the components can also be in two states. Binary  $K$  components out of  $n$ ,  $G$  system works only if at least  $k$  components work apparently. There is one  $K$  value with respect to a binary  $K$  out of  $n$ ;  $G$  system, or  $k$  out on  $n$ ;  $F$  system. The model of a generalized multistate  $K$  out of  $n$ ,  $G$  system and also of  $F$  system has been developed by [7], [8], [9], [11], [15], [16] and [17]. An exact evolution alongwith for multistate  $K$  out of  $n$  – system with the independent component is also obtained.

Some authors such that [3] discussed hot standby system and gave a comprehensive literature on survey of standby redundancy reliability analysis. Here in warm standby system reliability model when an operating component fails a standby component becomes active and repair begins of failed component

after repairing it is in active mode. If failure occurs when there is no one in active standby then all components are as good as total failure. Arulmozhi [3] has worked on such a model and gave good results.

Tian [2] obtained recursive formula by focusing on probability of the system in states below a certain state denoted by  $Q_{sd}$ . For performance evolution of multistate  $K$  out of  $n$  system he also obtained upper and lower bound of  $Q_{sd}$ . Dhillon et. Al. [6] discussed hot standby system.

Arulmozhi [2] have analyzed. Markovian parallel reliability system. Myer [11] designed to achieve very low probabilities of failure after use redundancy to meet these requirements. Cheng and Tang [5] extended one stage weighted  $K$  out of  $N$  model of two stage with components in common. He developed algorithm to calculate the reliability and generate the minimal cuts and minimal paths of two stage weighted  $K$  out of  $n$  systems. Arulmozhi [3] derived an equation for reliability of  $M$ -out of  $N$ -warm standby system with  $r$  service facilities to repair the failed component. This can be used in the calculation of the other system measures.

Reliability of  $M$ -out of  $N$ -warm standby system with service facility. Cold stand by and hot standby reliability systems are derived as particular cases of the model.

Here we are trying to use Rayleigh distribution to obtain the results for equation of reliability.

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**2.0 Mathematical Calculation**

Here the following assumptions are made as follows-

- a. The model is  $M$ -out of  $N$ -warm standby system. If  $N-M+1$  component fail, the system fails and at least  $M$  out of  $N$  components are required for the successful operation of the system.
- b. There are  $R$  repair facilities to repair the failed components ( $R \leq N$ )
- c. Each component has a constant operating failure rate and a
- d. constant standby failure rate and a state dependent exponential repair rate ... (1)
- e. There is perfect switching and sensing and that no warm up time is required.

Here the following notations are used.

- $N$ : Total number of components in the system
- $M$ : Minimum number of components required for the successful operation of the system.

$\lambda_o, \lambda_d$ : Failure rate of the operating and the standby components respectively.

$\mu$ : Repair rate of failed unit

$\mu_i, i = 1, 2, 3, \dots, R \dots (2)$

$R_o$ : at  $e^{-\frac{\lambda_o t^2}{2}}$  - reliability of an operating component

$R_d$ : at  $e^{-\frac{\lambda_d t^2}{2}}$  - reliability of a standby unit

$f_o$ :  $\lambda_o t e^{-\frac{\lambda_o t^2}{2}}$  - pdf of time of failure of an operating component, and

$f_d$ :  $\lambda_d t e^{-\frac{\lambda_d t^2}{2}}$  - pdf of time to failure of a standby unit.

$f_s, R_s$ : pdfs and reliability of the system respectively.

$t_i$ : Life time of the first component

( $i = 1, 2 \dots N-M-1$ ) and  $0 < t_{N-M} < t$

$y_i$ : Random variable pertaining to repair time of the  $i$ th component.

$x_i$ : Random variable pertaining to failure time of the  $i$ th component.

\*: Product of terms.

$\prod_{i=1}^{N-M-1} \int_{t_i}^{N-M-1}$ : Product of  $(N-M-1)$

$\prod_{i=1}^{N-M-1} dt_1 : dt_2, \dots, dt_{N-M}$

**The 1 Out of – 2 – Warm Standby System with Repair Facility**

Let us consider the simple case of 1 – out of 2 warm standby systems with one repair facility to repair the failed components. The reliability of a component with failure rate

$\lambda_c \geq 0$  is

$$R_s(t) = t \exp\left(\frac{-\lambda_c t^2}{2}\right) \text{ for } c = 0, d$$

Let us define the events  $E1$  and  $E2$  as

$E1$ : Occurs when either the operating component fails and the standby becomes operational, or the standby fails and the operational component remains operational.  $E2$ : Occurs when the remaining component fails and this happens before the service completion of the first failed component. In other words, service completion takes place only after time  $t$ . Thus we have

$$a. P[E_1] = \lambda_o t_1 e^{-\frac{\lambda_o t_1^2}{2}} t_1 e^{-\frac{\lambda_d t_1^2}{2}} dt_1 + t_1 e^{-\frac{\lambda_o t_1^2}{2}} \lambda_d t_1 e^{-\frac{\lambda_d t_1^2}{2}} dt_1 \dots (3)$$

$$b. P[E_2] = \lambda_o (t - t_1) e^{-\frac{\lambda_o (t-t_1)^2}{2}} p\left(\frac{T_1 > t}{X_2 < Y_1}\right) dt \dots (4)$$

Where,

$t_1$  is the life of first component ( $t - t_1$ ) is the operating life of other component.  $y_1$  is the repair time random variable of the  $i$ th component and  $x_1$  is the failure time random variable of the  $i$ th component.

$$P\left(y_1 > \frac{t-t_1}{x_2} < y_1\right) = \frac{P(Y_1 > t-t_1)}{P(X_2 < Y_1)} = \frac{\text{Exp}(-\mu_i(t-t_1))}{\left(\frac{\lambda_o}{\lambda_o + \mu_i}\right)} \dots (5)$$

$$f_s(t) = \int_0^t (\lambda_o + \lambda_c) \exp(-\lambda_o t_1) \lambda_o \exp[-(\lambda_o + \lambda_c)(t-t_1)] P\left(\frac{Y_1 > t}{X_2 < Y_1}\right) dt \dots (6)$$

Using (3), (4) and (5) in equation (6) and defining

$$R_s(t) = \int_t^\infty f_s(u) du$$

$$R_s(t) = [(\lambda_o + \lambda_c) + \lambda_d][\lambda_o + \lambda_c + \mu_i] \left[ \frac{\text{Exp}\{-(\lambda_o + \lambda_c) + \mu_i t\}}{\{(\lambda_o + \lambda_c) + \mu_i\}} \right] - \left[ \frac{\text{Exp}\{-(\lambda_o + \lambda_c) + \lambda_d t\}}{\{(\lambda_o + \lambda_c) + \lambda_d\}} \right]$$

When  $\mu_i = 0$  and  $\lambda_c = 0$  the above result reduce to reliability of 1- out – of - 2 system without repair facility, analyzed by Arulmozhi.

When  $\lambda_d = \lambda_o$  and  $\mu_i = 0$ , it reduced to active redundant system.

When  $\lambda_d = 0$  and  $\lambda_o = \lambda$  and  $\mu_i = 0$ , it reduces to cold standby system.

**The General  $M$  – Out –  $N$  Warm Standby System with Repair Facility**

Following methodology is given for 1 – out – of 2 warm standby models. The system reliability for the most general case is derived as follows

The pdf of the time of failure of the  $m$  – out – of –  $n$  warm standby system with  $r$  repair facility is given by

$$f_s(t, M, N) = \int_0^t \left[ M c_1 f_0(t_1)^{M-1} R_0(t)^{N-M} + N - M c_1 f_0(t_1) R_0(t)^{N-M-1} R_0(t)^M \right] \left\{ \frac{P(Y_1 > t)}{X_2 < Y_1} \right\} f_s(t - t_1, M, N - 1) dt_1 \quad \dots (7)$$

The system operating interval is divided into  $(N - M + 1)$  parts and if  $(N - M + 1)$  components are in service station, the system fails. System failure in  $dt$  after operating for time  $t$  is associated with  $(N - M + 1)$  simple events:-

- E1 : First component fails in  $dt_1$ , and the system operates.
- E2 : Second component fails in  $dt_2$ , and the probability of service completion of the first failed component is after time  $t$  and the system operates.
- EN – M :  $(N - M)$ th component fails in  $dtN - M$ , and the probability of service completion of the first failed  $(N - M - 1)$  components is after time  $t$  and the system operates.
- EN - M + 1 :  $(N - M + 1)$ th component fails in  $dt$ , and the probability of service completion of the first failed.  $(N - M)$  components are after time  $t$  and the system fails.

The second failed component may be an operating component or a standby unit.

$$P(x_2 < y_1) = \frac{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d}{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d + \mu_1} \quad \dots (8)$$

Now the pdf  $f_s(t, M, N)$  of the system is given by

$$f_s(t, M, N) = C_1^* \int_0^t \text{Exp}[-(M\lambda_0 t_1 + (N - M)\lambda_d t_1 + \mu_1(t - t_1))] f_s(t - t_1, M, N - 1) dt_1, \quad \dots (9)$$

Where

$$C_1 = \left[ \frac{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d}{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d + \mu_1} \right] [M c_1 \lambda_0 + N - M c_1 \lambda_d] \\ = C_2^* \int_0^t \int_0^{t_1} \text{Exp}[-(M\lambda_0 t_2 + \lambda_d t_1 - (N - M - 1)\lambda_d t_2 + \mu_1(t - t_1) + \mu(t - t_2))] f_s(t - t_2, M, N - 2) dt_1 dt_2$$

Where

$$C_2 = \int_0^t \left[ \frac{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d}{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d + \mu_1} \right] [M c_1 \lambda_0 + N - M + 1 - i c_1 \lambda_d]$$

Continuing the expansion of  $f_s(t - t_2, M, N - 2)$

We get  $f_s(t, M, N) dt$  as

$$f_s(t, M, N) = C_{N-M} \int_0^t \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \text{Exp}[-(M\lambda_0 t$$

$$- \sum_{k=1}^{N-M} (\lambda_d t_k + \mu_k(t - t_k))] \prod_{i=1}^{N-M} dt_i \quad \dots (10)$$

Where

$$C_{N-M} = \prod_{i=1}^{N-M} \left[ \frac{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d}{M(\lambda_0 + \lambda_c) + (N - M - 1)\lambda_d + \mu_i} \right] [M c_1 \lambda_0 + N - M + 1 - i c_1 \lambda_d] \quad \dots (11)$$

And  $f_s(t - t_2, M, N - 2)$  reach the bound.

Keeping the view of (10) and (11) we consider the following term

$$\int_0^t \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \text{Exp} \left[ -(M\lambda_0 t - \sum_{k=1}^{N-M} (\lambda_d t_k + \mu_k(t - t_k))) \right] \prod_{i=1}^{N-M} dt_i \quad \dots (12)$$

From (10) it is equal to

$$= \int_0^t \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \text{Exp} \left[ -(M\lambda_0 t - \sum_{k=2}^{N-M} (\lambda_d t_k + \mu_k(t - t_k))) \right] \prod_{i=1}^{N-M} dt_i \int_0^{t_1} \text{Exp}[-\lambda_d t_1 - \mu_1(t - t_1)] dt_1 \\ = \int_0^t \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \int_0^{t_1} \text{Exp} \left[ -(M\lambda_0 t - \sum_{k=2}^{N-M} (\lambda_d t_k + \mu_k(t - t_k)) - \sum_{k=1}^1 \mu_k t) \right] \prod_{i=1}^{N-M} dt_i \int_0^{t_1} \text{Exp}[-\lambda_d t_1 - \mu_1(t - t_1)] dt_1 \\ = \int_0^t \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \int_0^{t_1} \int_0^{t_2} \text{Exp} \left[ -(M\lambda_0 t - \sum_{k=2}^{N-M} (\lambda_d t_k + \mu_k(t - t_k)) - \sum_{k=1}^2 \mu_k t) \right] \prod_{i=1}^{N-M} dt_i \int_0^{t_1} \int_0^{t_2} \text{Exp}[-2\lambda_d t_2 - (\mu_1 + \mu_2)t_2] dt_2 - \int_0^{t_1} \int_0^{t_2} \text{Exp}[-(\lambda_d - \mu_1)t_2] dt_2$$

In general it is equal to

$$= \int_0^t \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \text{Exp} \left[ -(M\lambda_0 t - \sum_{k=1}^{N-M} (\lambda_d t_k + \mu_k(t - t_k)) - \sum_{k=1}^i \mu_k t) \right] \prod_{i=1}^{N-M} dt_i \int_0^{t_1} \dots \int_0^{t_{N-M-1}} \left[ \frac{\text{Exp}[-(i\lambda_d - \sum_{k=1}^i \mu_k)t_i(-1)^{i-1}]}{\prod_{k=1}^{i-1} [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} \right] \\ + \frac{\text{Exp}[-(i-1)\lambda_d - \sum_{k=2}^i \mu_k)t_i(-1)^{i-1}}{\prod_{k=1}^{i-2} [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} \prod_{k=2}^{i-1} [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} + \frac{\text{Exp}[-(\lambda_d - \mu_1)t_i(-1)^{i+1}]}{\prod_{k=1}^{i-1} [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} \quad \dots (13)$$

$$= \frac{\text{Exp}[-(M\lambda_0 + (N - M)\lambda_d)t](-1)^{N-M}}{\prod_{k=1}^{N-M} [k\lambda_d - \sum_{m=1}^k \mu_m]} \\ + \frac{\text{Exp}[-(M\lambda_0 + (N - M)\lambda_d)t](-1)^{N-M-1}}{\prod_{k=1}^{N-M-1} [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} \prod_{k=1}^1 [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} \\ + \frac{\text{Exp}[-(M\lambda_0 + (N - M - 2)\lambda_d + (\mu_1 + \mu_2))t](-1)^{N-M-2}}{\prod_{k=1}^{N-M-2} [k\lambda_d - \sum_{m=1}^{k-2} \mu_m]} \prod_{k=1}^2 [k\lambda_d - \sum_{m=1}^{k-1} \mu_m]} + \dots \\ + \frac{\text{Exp} \left[ -(M\lambda_0 + \sum_{m=1}^{N-M} \mu_m)t \right]}{\prod_{k=1}^{N-M} [k\lambda_d - \sum_{m=1}^{N-M-k} \mu_m]} \quad \dots (14)$$

$$= \sum_{i=0}^{N-M} \frac{\text{Exp} \left[ -M\lambda_0 t - i\lambda_d t - \sum_{j=1}^{N-M-i} \mu_j t \right] (-1)^i}{\prod_{i=0}^{N-M-i} [k\lambda_d - \sum_{m=N-M-i}^{N-M-i+k} \mu_m]} \prod_{k=1}^i [k\lambda_d - \sum_{m=N-M+i-1}^{N-M+k-1} \mu_m]} \quad \dots (15)$$

$$R_s(t) = \prod_{j=0}^{N-M} \left[ \frac{M(\lambda_0 + \lambda_j) + (N-M-j)\lambda_j}{M(\lambda_0 + \lambda_j) + (N-M-j)\lambda_j + \mu_j} \right] [Mc_1\lambda_0 + (N-M+1-j)c_1\lambda_j]^*$$

$$\int_0^{\infty} \sum_{i=0}^{N-M} \left[ \frac{\text{Exp}[-M\lambda_0 t - i\lambda_j t - \sum_{j=1}^{N-M-i} \mu_j t] (-1)^i}{\prod_{k=1}^{N-M-i} [k\lambda_0 - \sum_{m=N-M-i+1}^{N-M-i+k} \mu_m]} \prod_{k=1}^i [k\lambda_0 - \sum_{m=N-M-i+1}^{N-M-i+k} \mu_m]} \right] dt$$

$$R_s(t) = \prod_{j=0}^{N-M} \left[ \frac{M(\lambda_0 + \lambda_j) + (N-M-j)\lambda_j}{M(\lambda_0 + \lambda_j) + (N-M-j)\lambda_j + \mu_j} \right] [Mc_1\lambda_0 + (N-M+1-j)c_1\lambda_j]^*$$

$$\sum_{i=0}^{N-M} \left[ \frac{\text{Exp}[-M\lambda_0 t - i\lambda_j t - \sum_{j=1}^{N-M-i} \mu_j t] (-1)^i}{\left[ -M\lambda_0 - i\lambda_j - \sum_{j=1}^{N-M-i} \mu_j \right]^* \prod_{k=1}^{N-M-i} \left[ k\lambda_0 - \sum_{m=N-M-i+1}^{N-M-i+k} \mu_m \right] \prod_{k=1}^i \left[ k\lambda_0 - \sum_{m=N-M-i+1}^{N-M-i+k} \mu_m \right]} \right] \dots (16)$$

In (16) if we take  $\mu_j = 0$  it reduces to the reliability without repairman case as discussed by Arulmozhi and is given by:

$$R_s(t) = \sum_{i=0}^{N-M} \frac{(-1)^i}{i!(N-M-i)!} \prod_{j=0}^{N-M} [M(\lambda_0 + \lambda_j) + j\lambda_j] \text{Exp}[-(M\lambda_0 + i\lambda_j)t] \dots (17)$$

For finding the different values of  $R_s(t)$  at  $\lambda_c = 0.2$ , we will put  $N = 7, M = 3, \lambda_0 = 1, \lambda_d = 0.5, t = 0$  to 5 in (15)

$$R_s(t) = \sum_{i=0}^4 \frac{(-1)^i}{i!(4-i)!(0.5)^4} \prod_{j=0}^4 (3.6 + 0.5j) e^{-(3+0.5i)t}$$

$$R_s(t) = \frac{16}{4!} \prod_{j=0}^4 (3.6 + 0.5j) e^{-3t} - \frac{16}{3!} \prod_{j=0}^4 (3.6 + 0.5j) e^{-3.5t} + \frac{16}{2!2!} \prod_{j=0}^4 (3.6 + 0.5j) e^{-4t} - \frac{16}{3!} \prod_{j=0}^4 (3.6 + 0.5j) e^{-4.5t} + \frac{16}{4!} \prod_{j=0}^4 (3.6 + 0.5j) e^{-5t}$$

$$R_s(t) = \frac{16}{24} (4.1 \times 4.6 \times 5.1 \times 5.6) e^{-3t} - \frac{16}{6} (3.6 \times 4.6 \times 5.1 \times 5.6) e^{-3.5t} + \frac{16}{4} (3.6 \times 4.1 \times 5.1 \times 5.6) e^{-4t} - \frac{16}{6} (3.6 \times 4.1 \times 4.6 \times 5.6) e^{-4.5t} + \frac{16}{24} (3.6 \times 4.1 \times 4.6 \times 5.1) e^{-5t}$$

$$R_s(t) = 359.09 e^{-3t} - 1261.21 e^{-3.5t} + 1686.18 e^{-4t} - 1013.91 e^{-4.5t} + 230.85 e^{-5t} \text{ {at } t=0}$$

$$R_s(t) = 359.09 - 1261.21 + 1686.18 - 1013.91 + 230.85 = 1 \text{ {at } t=1}$$

$$R_s(t) = 359.09 e^{-3} - 1261.21 e^{-3.5} + 1686.18 e^{-4} - 1013.91 e^{-4.5} + 230.85 e^{-5} = 17.88 + 30.88 + 1.56 - 38.08 - 11.26 = 0.98 \text{ {at } t=2}$$

$$R_s(t) = 359.09 e^{-6} - 1261.21 e^{-7} + 1686.18 e^{-8} - 1013.91 e^{-9} + 230.85 e^{-10} = 0.89 + 0.56 + 0.01 - 1.15 - 0.13 = 0.18 \text{ {at } t=3}$$

$$R_s(t) = 359.09 e^{-9} - 1261.21 e^{-10.5} + 1686.18 e^{-12} - 1013.91 e^{-13.5} + 228.54 e^{-15} = 0.0438 + 0.0103 + 0.00006 - 0.0346 - 0.00138 = 0.018 \text{ {at } t=4}$$

$$R_s(t) = 359.09 e^{-12} - 1261.21 e^{-14} + 1686.18 e^{-16} - 1013.91 e^{-18} + 230.85 e^{-20} = 0.002 - 0.00105 + 0.000180 - 0.0000154 + 0.00000047 = 0.0012 \text{ {at } t=5}$$

$$R_s(t) = 359.09 e^{-15t} - 1261.21 e^{-17.5t} + 1686.18 e^{-20t} - 1013.91 e^{-22.5t} + 230.85 e^{-25t} = 0.0001098 - 0.00003167 + 0.00000347 - 0.00000017 + 0.000000032 = 0.000081$$

For fixed values of  $M, N, \lambda_0, \lambda_d, t$  we obtain the values of  $R_s(t)$  as follows-

**Table 1:**

t	$R_s(t)$					
	0	1	2	3	4	5
$\lambda_c = 0.2$	1	0.98	0.18	0.018	0.0012	0.000081
$\lambda_c = 0.02$	1	0.68	0.12	0.011	0.0008	0.00005
$\lambda_c = 0.05$	1	0.72	0.13	0.012	0.0009	0.00005
$\lambda_c = 0.08$	1	0.76	0.14	0.013	0.0009	0.00006

In many practical situations, an  $M - out - of - N$  configuration serves as a useful system. As an example of this structure, consider systems with  $N$  independent and identically distributed components. The components could be communication channels processing and transmitting messages, or even production lines in a factory. If the number of components falls below  $M$ , the system fails. We find that communication channels or production lines can fail even they are kept standby so this type of example which we dealt is more useful.

From the Table 1 and figure -1, we observe that the reliability increases with the increase in numerical value of  $c$  but  $c$  should not be more

than 1; also we find that reliability decreases with the increase of time.

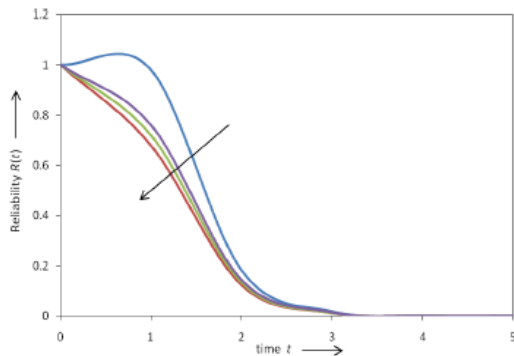


Fig -1 : Reliability for different value of  $\lambda_0$

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